Mathematical Meaning-Making and its Relation to Design of Teaching

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This paper addresses the design of teaching to promote engineering students' conceptual understanding of mathematics, and its outcomes for mathematical meaning-making. Within a developmental research approach, inquiry-based tasks have been designed and evaluated, through the use of competencies proposed for their potential to promote conceptual learning. A sociocultural frame draws attention to interactions between different cultural elements to address challenges to teaching related to student perspectives and the mathematical meanings they develop. The paper recognizes tensions between design of inquiry-based practice and the outcomes of that practice, and demonstrates the need for new research to address mathematical meanings of a student community within a sociocultural frame.

Keywords: Functions; Mathematical meaning; Teaching

Creación de significado matemático y su relación con el diseño en la enseñanza

En este trabajo se aborda el diseño de la enseñanza para promover la comprensión conceptual de las matemáticas por parte de estudiantes de ingeniería, y sus resultados para crear significado matemático. Dentro de un enfoque de investigación del desarrollo, las tareas se han diseñado y evaluado a través del uso de competencias propuestas por su potencial para promover aprendizaje conceptual. Un marco sociocultural llama la atención sobre las interacciones entre los diferentes elementos culturales para hacer frente a los retos de la enseñanza en relación a las perspectivas de los estudiantes y los significados matemáticos que desarrollan. El artículo reconoce las tensiones entre el diseño de la práctica basada en la investigación y los resultados de esa práctica, y demuestra la necesidad de nuevas investigaciones para abordar los significados matemáticos de una comunidad estudiantil dentro de un marco sociocultural.

Términos clave: Enseñanza; Funciones; Significado matemático

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In this paper I focus on the Engineering Students Understanding Mathematics (ESUM) project (Jaworski & Matthews, 2011) in which an innovation in the teaching of a basic mathematics module to first year engineering students (n = 48) was studied. The ESUM innovation involved design of teaching using inquiry-based tasks and small group activity within a GeoGebra environment. The goal of teaching development was to promote students' conceptual understanding of mathematics rather than understanding that is instrumental or procedural (Hiebert, 1986; Skemp, 1976). The etymology of understanding (under-standing), as for example in comparison with the French word *comprendre* (taking together), is of interest. We have been challenged to declare what we mean by understanding or indeed by conceptual understanding and how we expect to recognize it. One finding from ESUM was the difficulty of discerning students' conceptual understanding, which we have expressed in terms of students' mathematical meaning making. We want to go beyond superficial indicators (like test or exam results) to find ways of revealing students' mathematical meanings. The focus of this paper is how to address this challenge. I consider a (draft) report from the Société Européenne pour la Formation des Ingénieurs (SEFI) Mathematics Working Group (2012) which recommends a competence approach (Niss, 2003). More precisely, this paper investigates the following research questions:

- When a (developmental research) project seeks to enhance students' meaning making of mathematics, how can we gain insights to students' mathematical meanings?
- How can we characterize mathematical meaning making in ways which aid its creation? (In what ways can the SEFI/Niss competence framework aid characterization?)

To address these questions, I draw on findings from ESUM, using the SEFI framework to interrogate the design of inquiry-based tasks or questions, taking, as an example, the topic of functions, which was a central topic in the mathematics module.

A literature search on innovative modes of teaching (in HE STEM-related subjects[1]) showed that the use of inquiry-based approaches is often conceptualized within a constructivist theoretical frame (Abdulwahed, Jaworski, & Crawford, 2012). As such, learning is considered from individual cognitive perspectives, possibly with a social dimension (e.g., Ernest, 1991). In ESUM, research findings have pointed to tensions and contradictions between the design of teaching and students' perspectives on learning and teaching (Jaworski, Robinson, Matthews, & Croft, 2012). This has required us to deal with complexity within differing cultures and within institutional constraints, for which a sociocultural theoretical frame makes more sense than a frame of individual cognition. Thus, we see mathematics knowledge growing in social set-

[1] STEM (Science, Technology, Engineering and Mathematics). The ESUM project was funded through the Higher Education (HE) STEM programme by the Royal Academy of Engineering. Two case studies from the project can be found at http://www.hestem.ac.uk/resources/case-studies.

tings through mediational processes and the use of tools such as inquiry-based tasks and approaches to teaching (Schmittau, 2003; Wertsch, 1991).

A Developmental and Inquiry-Based Approach

The ESUM study employed a developmental methodology, incorporating an inquiry-based approach, in which research both studied developmental practice and contributed to development (Jaworski, 2003). A team of three teacher-researchers (insiders) designed and taught the module, with continuous reflection and review leading to modifications during practice and new insights for the next year of teaching. A research assistant (outsider) collected data and analyzed data together with the teaching team. Analyses informed future teaching.

The developmental methodology involving nested layers of inquiry (A, B & C with $A \subset B \subset C$) with students' learning of mathematics at the center: Inquiry in mathematics (A) involves students in learning and understanding mathematics through inquiry. Inquiry in developing mathematics teaching (B), involves questioning teaching approaches and the design of teaching, to understand the basis of teaching decisions and ways of improving teaching for better learning outcomes. Inquiry in layer C inspects the other two levels to gain insights to the developmental processes in both layers, and their outcomes (Jaworski, 2006). When inquiry practices are instituted or promoted within a group, an outcome can be the formation of an inquiry community, which can be seen to have all the hallmarks of a community of practice, as designated by Wenger (1998), except in one major respect. In Wenger's terms, those involved in the community can be seen to have joint engagement, enterprise and repertoire; and their identities can be conceptualized as encompassing the use of imagination in charting personal trajectories of engagement, and alignment with the norms and expectations within the practice (Wenger, 1998). While it is impossible to be a part of a community of practice without aligning with its norms and expectations, one does not have to align uncritically. Uncritical alignment can result in perpetuation of practices which do not achieve the goals of practitioners—for example, alignment with certain forms of teaching practice, within a community of mathematics teaching, can result in student learning outcomes which are instrumental or procedural in nature, lacking conceptual depth (Hiebert, 1986; Skemp, 1976). So, alignment needs to be critical—critical alignment— in which (established) practices are subject to critical questioning by the practitioners who engage with them (Jaworski, 2006). In learning mathematics, with inquiry in the three layers A, B and C critical alignment involves asking why? Why do we do things in certain ways: why this formula, why this procedure, why these relationships? Inquiry-based tasks and questions are designed to get student to address these whys.

Levels of Competency in Mathematics and in Teaching

I turn now to competence and competency and their relation to the design and use of inquiry-based questions and tasks to address the given research questions and the ESUM main goal regarding conceptual understanding. Niss (2003) writes "Possessing mathematical competence means having knowledge of, understanding, doing and using mathematics and having a well-founded opinion about it, in a variety of situations and contexts where mathematics plays or can play a role" (p. 183).

A mathematical competency is a distinct major constituent in mathematical competence: Eight competencies have been identified in two groups (SEFI, 2012).

The ability to ask and answer questions in and with mathematics. (1) Thinking mathematically, (2) reasoning mathematically, (3) posing and solving mathematical problems, and (4) modelling mathematically.

The ability to deal with mathematical language and tools. (5) Representing mathematical entities, (6) handling mathematical symbols and formalism, (7) communicating in, with and about mathematics, and (8) making use of aids and tools.

These competencies seems to have synergy with inquiry-based learning and what we aimed for in the ESUM project, and they offer starting points for the design of tasks and evaluation of learning outcomes. Space here precludes a detailed account of each competency. I will rather clarify their meaning through application to task design. The authors emphasize three dimensions for specifying and measuring progress in learning with respect to competency: (a) degree of coverage: The extent to which the person masters the characteristic aspects of a competency; (b) radius of action: The contexts and situations in which a person can activate a competency; and (c) technical level: How conceptually and technically advanced the entities and tools are with which the person can activate the competence. How to address these dimensions is an issue to consider.

Task Design and Analysis

When students emerge from schools and their A level[2] courses, we know that their mathematical learning has often been of an instrumental nature (e.g., Artigue, Batanero, & Kent, 2007; Hernández-Martínez, Williams, Black, Pampala, & Wake, 2011). Thus, as part of the school culture, they know how to, for example, apply rules of differentiation and integration, but have little conceptual understanding of the nature of functions or of limiting processes. In both of these areas research has pointed to conceptual difficulties that students experience (e.g., Cornu, 1991; Even & Tirosh, 1995). So, the demands of design within the university course are to create tasks which en-

[2] A level course means courses preparing students for Advanced Level General Certificate of Education, a public examination qualifying students for study in Higher Education. These are high stakes examinations and schools are measured by their examination successes.

gage students with mathematics, some of which is already familiar to them, in ways which take them beyond school practices and into a university culture in which it is hard to progress without deeper understandings.

The following two tasks (see Table 1) were designed for these purposes. The first was used in a lecture at the beginning of our work on functions. In the second, the first part (a) was used in a lecture and the other parts (d-e) in a tutorial where students sat in groups of three or four each with a computer and access to GeoGebra software. In accord with design goals, and associated expectations of students' engagement, I have analyzed the tasks in terms of the eight mathematical competencies previously mentioned.

Analysis of Task 1 Using the Competencies

Task 1 (Table 1) was intended to open up discussion of functions. The lecturer offered the task and waited for students to write down two functions, meanwhile, walking round the lecture theatre and looking expectantly at students (and smiling, with eye contact) to encourage their engagement with the task.

Table 1
Two Tasks from ESUM, with Associated Competencies

		Task	Competencies
1		Think about what we mean by a function and write down two examples. Try to make them different examples.	1, 2, 5, 6, 7
2		In the topic area of real valued functions of one variable. Consider the function $f(x) = x^2 + 2x$ (x is real).	
	a)	Give an equation of a line that intersects the graph of this function. (i) Twice (ii) Once (iii) Never (Adapted from Pilzer, Robinson, Lomen, Flath, Hughes Hallet, Lahme, et al., 2003, p. 7)	1, 2, 3, 5, 6, 7
	b)	If we have the function $f(x) = ax^2 + bx + c$. What can you say about lines which intersect this function twice?	1, 2, 7 (, 8)
	c)	Write down equations for three straight lines and draw them in GeoGebra.	1, 2, 5, 6, 7, 8
	d)	Find a (quadratic) function such that the graph of the function cuts one of your lines *twice*, one of them *only once*, and the third *not at all* and show the result in GeoGebra.	1, 2, 3, 7, 8
	e)	Repeat for three different lines (what does it mean to be different?).	1, 2, 3, 5, 6, 7 (, 8)

The task is open in nature. Students could write down any example they could think of. Since most students had studied A level mathematics, they had certainly encountered the term function and used functions. So for most students the task was accessi-

ble. It encouraged them to think (competence 1). To write down the function they had to use symbolism to represent the function (competences 5 and 6). I argue that in writing down, they were already starting to communicate, and, in deciding on different functions, to reason mathematically (competences 7 and 2). After a suitable time, the lecturer, in plenary, asked students to offer one of the functions they had written, and wrote these verbatim on the overhead projector. Initial contributions were made tentatively, the lecturer smiling encouragement and thanking the student, and many more then followed, thus overcoming some of the barriers to student contribution in a lecture. When a (long) list of offerings had been produced, the lecturer asked students to comment on the nature of what had been offered (importantly, a student who had offered any example was now anonymous). Some of the examples offered were as follows: $y = x + 3$; $y = x^2$; $y = e^x$; $x + y = 4$; $f(x) = x + 1$. The majority was of the form $y =$. When asked to comment on difference some students mentioned linear functions versus quadratic functions, or exponential functions. Some queried $x + y = 4$, stating that it is an equation, not a function. Very few used functional notation of the form $f(x) =$. When the lecturer added to the list $y = 5$ and $x = 4$, students were adamant that these are not functions. Thus, communication occurred between students and the lecturer (competence 7), and students offered explanations and reasons for why an item was a function or not (competence 2). Students could see alternative offerings from their peers. For the lecturer, students' responses to the task provided insights to their current knowledge/thinking about functions, and allowed some immediate challenge—for example, "what is the difference between a function and an equation?", "why do you think $y = 5$ and $x = 4$ are not functions?"

In Task 1, students had to produce their own examples, leading to engagement, questioning, discussion and inquiry. Inquiry could be seen in the questioning which resulted, in consideration of what is a function and what is not a function, and in the mode of engagement in the lecture: Students were expected to contribute, think, reason, argue, not to take some things for granted, and to deal with uncertain situations (not everything will be presented as cut and dried, right or wrong). We can see this episode as the beginnings of creating an inquiry community. We see here some starting points in addressing the first research question, and a start to characterization of understanding using the competencies (research question 2). We can ask how the three dimensions relating to competency can be used to evaluate students' responses to the task.

Analyzing Task 2 Using the Competencies

The first part of Task 2 (2a) was also presented in a lecture with a similar teaching approach to that described above. An analysis of this task suggests that:

- The function is easy to sketch for students who have reached A level in school —it is easy to see lines which cross it in the three conditions (competence 5).
- Students have to talk to each other (competence 7).

- ♦ They have to think about equations for their lines (competences 1, 3, and 6).
- ♦ They start to reason about the differences between the lines (competence 2).
- ♦ They have to give feedback to the lecturer and others in the cohort (competences 2 and 7).

In the lecture, students were asked to write down the required equations and to discuss with a neighbor (competences 1, 5, 6, and 7). After a short time, students' suggestions were written on the overhead projector by the lecturer. Some students offered equations of parallel horizontal lines, such as $y = 1$, $y = -1$, and $y = -3$. Others offered non-horizontal lines. One question which arose was how one can know that a non-horizontal line will cross the graph (or not). This provided opportunity for discussion, with some students disagreeing with others as to which lines will cross or not cross (competences 1, 2, and 3). Further graphical and algebraic activity resulted. GeoGebra allowed the possibility to experiment quickly changing coefficients in equations and scales on axes to gain insights into relationships. Some students were able to offer algebraic reasoning, but it was not certain that all were able to understand this (competence 6).

The above analysis relates to layer A of our developmental methodology: It looks critically at the ways in which mathematics is offered to students in inquiry-based approaches and the opportunities it provides for their learning and understanding. In layer B we address the lecturer's learning from inquiring into the teaching approach and its outcomes. Here the lecturer learns from students' responses and can consider how to plan differently for a future occasion, to give more time or not, to rearrange material or not. The lecturer also learned about interventions: Where a question or explanation seemed to promote student engagement and where not; how to deal with incorrect assertions if no student offered a challenge. When students' themselves offered a challenge, mathematical communication between students provided corresponding opportunities for learning. The lecturer became aware of actions which promoted or inhibited students from offering such challenges. More time could valuably have been spent on such activity, encouraging questions and explanations from students, but there was much further material to address in the lecture, and so not enough time to give to continuing the discussion. These are examples of contextual constraints.

Tasks for Use in a Tutorial

Parts b to e of Task 2 were addressed in a tutorial. Typically a tutorial was related to material addressed in a recent lecture. Often a sheet of questions was provided, some questions offered practice and support in relevant areas of mathematics; others were inquiry-based questions in which exploration, questioning, discussion and justification were encouraged. Task 2 is an example of the latter: Part b requires students to generalize from a (competencies 1, 2, and 7); in part c students have to invent their own mathematical objects and use a technological tool (competences 1, 2, 5, 6, 7, and 8); in part d they have to tackle an open-ended problem (competences 1, 2, 3, 7, and 8),

and in part e they are required to generalize mathematically (competences 1, 2 3, 5, 6, and 7). In parts b and c, use of GeoGebra can provide opportunities to visualize and to generate a range of possibilities for consideration. Part d is seriously challenging —even with use of GeoGebra it is not simple to generate the function, analytical thinking is required. In part e, the question about "what it means to be different" is designed to promote thinking at more general levels and encourages movement towards conjecture and proof.

In the tutorials, lecturer and a graduate assistant moved from group to group of students encouraging work on tasks and probing students' mathematical thinking. It became clear that different groups engaged very differently: some taking on the mathematical challenges and some seeking quick and easy solutions. GeoGebra was used variously as a graphical display (with a screen full of indistinguishable graphs), a source of quick/easy answers to questions, or as a help in tackling challenging questions. While tutor and assistant encouraged the latter, they were aware of the other uses. Although their questions encouraged a more meaningful, mathematically in-depth use, it could be seen, when the tutor left the group, that some students returned to other uses or were tempted to use social networking sites or engage with email. Critical alignment for the tutor is seen in how to promote deeper engagement when former school practice and current student cultures acted in other directions.

Data and Analysis

Data collected from these events included the lecturer's reflections: orally after a lecture or tutorial, and a written reflection each week addressing issues arising from the interpretation of teaching design in practice (critical alignment); the research assistant audio-recorded lectures and the oral reflections and kept observational notes from all events. After the end of the module (one semester), she and another colleague interviewed a selection of students. In addition, data was collected from student surveys and written project work. Data were analyzed to address questions of students' engagement and their experiences of inquiry-based tasks and use of GeoGebra. Data from written project work showed that students were aware of ways in which GeoGebra could contribute to their understanding. However, the following two responses, from focus group interviews, are indicative of student attitudes.

- I found GeoGebra almost detrimental because it is akin to getting the question and then looking at the answer in the back of the book. I find I can understand the graph better if I take some values for x and some values for y, plot it, work it out then I understand it…if you just type in some numbers and get a graph then you don't really see where it came from. (Focus group 1)

- Understanding maths—that was the point of Geogebra, wasn't it? Just because I understand maths better doesn't mean I'll do better in the exam. I have done less past paper practice. (Focus group 2)

How dimensions of competency might interface with such findings is hard to see. The above discussion relates to layer C of our developmental methodology, focusing on the outcomes from inquiry in layers A and B. The final section below looks critically at the analyses in all three layers.

MAKING SENSE OF STUDENT UNDERSTANDING

In the above I have focused on analyses of the design of teaching, principally the design of inquiry-based tasks and an associated teaching approach to engage students with mathematics for conceptual understanding. I have used mathematical competencies to qualify or start to characterize conceptual understanding. I have suggested that students' responses to this careful design have not been what we would ideally have liked; factors identified being institutional constraints, time, students' school culture, students' social culture. Student remarks such as those quoted led us to characterize student responses as strategic (Jaworski & Matthews, 2011). Students wanted the best possible grades and had clear ideas as to how this should be achieved. Some of these ideas conflicted with the expectations of teaching, students expressed their own expectations on the nature of teaching (e.g., the teaching should focus more on graph plotting; there should be time given to practicing past papers). Comments related to doing "better in the exam" suggested students valuing a more instrumental approach to understanding with a perception that a more in-depth understanding was unnecessary.

We are aware that the existence and nature of an exam (worth 60%), whose style had changed little from that before the innovation, was not exactly in the spirit of inquiry-based learning to encourage deeper understanding, although it might be seen to have synergy with dimensions of competency. We have considered replacing the exam with other forms of assessment, but institutional constraints have so far prevented this[3]. In written group project reports (worth 20%), understanding was demonstrated through responses to questions in which students had to pursue their own lines of inquiry and comment on the value of their use of GeoGebra. A typical response was the following one from a project report written by one small group:

> *As a group we looked at many different functions using GeoGebra and found that having a visual representation of graphs in front of us gave a better understanding of the functions and how they worked. In this project the ability to be able to see the graphs that were talked about helped us to spot patterns and trends that would have been impossible to spot without the use of GeoGebra.*

[3] It is ironic that, in the exam at the end of the ESUM innovation, students' scores were on average 10% higher than those of previous cohorts. We are not able to link this directly to the innovation, since data was not available to compare intake grades with those of previous cohorts.

However, observational data showed some students not engaging seriously with the more demanding questions in tutorials, and many attending more assiduously to the more routine exercises. It was clear that where groups were taking seriously the inquiry-based questions, discussion with the tutor proved encouraging and motivating. Unsurprisingly, groups which responded best in tutorials gained the higher marks in the assessed group project.

In the above I have commented briefly on some of the key findings from our ESUM analyses. They reveal important insights into the sociocultural factors influencing the implementation of project design and its outcomes for students. Nevertheless, the nature of mathematical understanding remains elusive. Analyses using the competency framework have supported our design of tasks; apparent synergy between principles of inquiry and competency reinforce confidence in our didactic design. However, our research questions above are only partially addressed. The competency-based task analysis offers a form of characterization (Task 2, part a). The sociocultural analyses allow us to frame some of the obstacles to deeper insights into students' understanding (e.g., students' perceptions demonstrated in project writing in comparison to their views expressed orally in interview). The competencies and dimensions offer a framework for the design and evaluation of tests or examinations, but we believe this would give us little more than a summative evaluation of the sort we have already from exam and test scores, albeit perhaps more detailed and specific. The second research question—How can we characterize understanding in ways which aid its creation?—is only partially addressed, and perhaps we need a better-focused question. What is it, exactly, that we are trying to characterize? So far we have reinforced our design principles and the elusive nature of discerning students' mathematical understanding. We have juxtaposed design principles with sociocultural findings using activity theory to highlight inherent tensions or conflicts (Jaworski et al., 2012). Discerning tensions and conflicts is one step towards resolving them. Finding ways to characterize understanding is another. We still need to make the sociocultural findings active in our design so that we come closer to enabling the student understandings we seek. This requires us to go beyond competencies, while remaining aware of their contribution towards recognition of the mathematics for which we seek understanding. Since these are engineering students, discussion is taking place also with the engineering department.

A final consideration is methodology. We need to adapt our methodological approach with a deeper focus on mathematical meanings. The use of questions in lectures could be more focused towards revealing meanings. This would require the lecturer to pay more attention to generating student articulation of meanings which would be recorded and analysed within the sociocultural frame of the lecture community. We are designing further research to address these considerations.

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